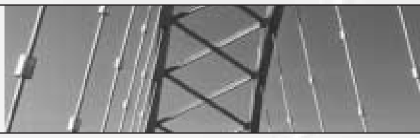


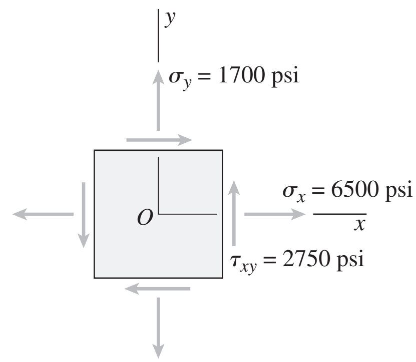
7

Analysis of Stress and Strain

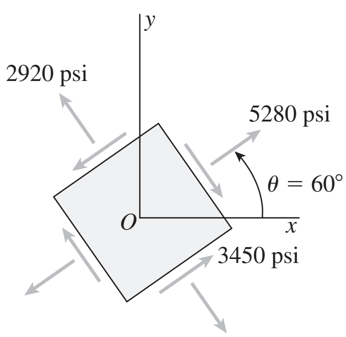
Plane Stress

Problem 7.2-1 An element in *plane stress* is subjected to stresses $\sigma_x = 6500$ psi, $\sigma_y = 1700$ psi, and $\tau_{xy} = 2750$ psi, as shown in the figure.

Determine the stresses acting on an element oriented at an angle $\theta = 60°$ from the x axis, where the angle θ is positive when counterclockwise. Show these stresses on a sketch of an element oriented at the angle θ.



Solution 7.2-1 **Plane stress (angle θ)**



$\sigma_x = 6500$ psi $\quad \sigma_y = 1700$ psi $\quad \tau_{xy} = 2750$ psi

$\theta = 60°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

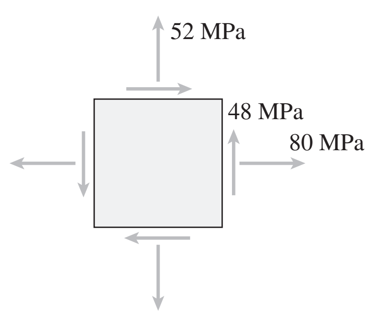
$$= 5280 \text{ psi} \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

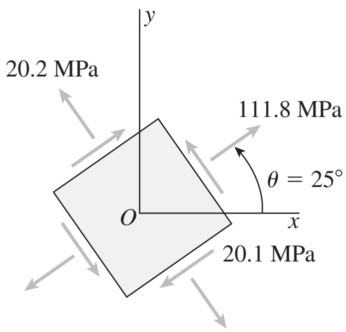
$$= -3450 \text{ psi} \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 2920 \text{ psi} \longleftarrow$$

Problem 7.2-2 Solve the preceding problem for $\sigma_x = 80$ MPa, $\sigma_y = 52$ MPa, $\tau_{xy} = 48$ MPa, and $\theta = 25°$ (see figure).



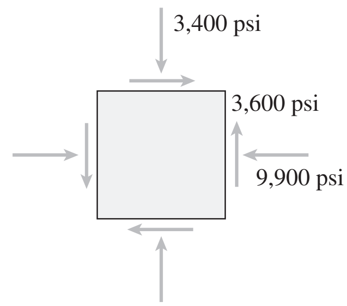
Solution 7.2-2 **Plane stress (angle θ)**



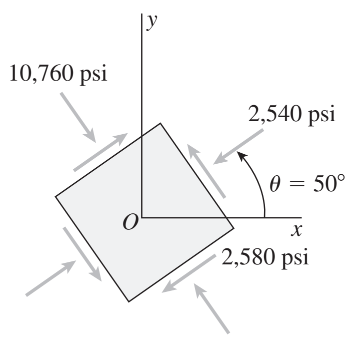
$\sigma_x = 80\text{ MPa} \quad \sigma_y = 52\text{ MPa} \quad \tau_{xy} = 48\text{ MPa}$
$\theta = 25°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= 111.8\text{ MPa} \longleftarrow$$
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= 20.1\text{ MPa} \longleftarrow$$
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 20.2\text{ MPa} \longleftarrow$$

Problem 7.2-3 Solve Problem 7.2-1 for $\sigma_x = -9{,}900$ psi, $\sigma_y = -3{,}400$ psi, $\tau_{xy} = 3{,}600$ psi, and $\theta = 50°$ (see figure).



Solution 7.2-3 **Plane stress (angle θ)**

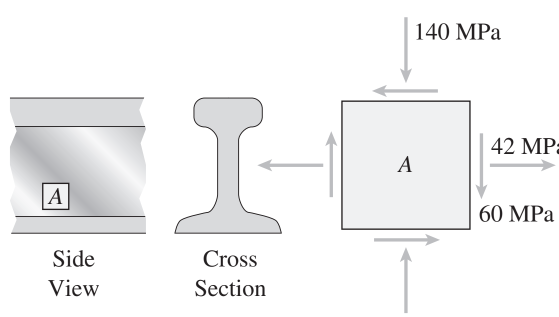


$\sigma_x = -9900\text{ psi} \quad \sigma_y = -3400\text{ psi} \quad \tau_{xy} = 3600\text{ psi}$
$\theta = 50°$

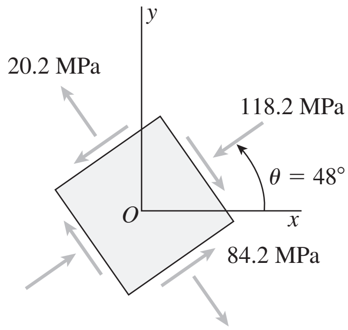
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= -2540\text{ psi} \longleftarrow$$
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= 2580\text{ psi} \longleftarrow$$
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -10{,}760\text{ psi} \longleftarrow$$

Problem 7.2-4 The stresses acting on element *A* in the web of a train rail are found to be 42 MPa tension in the horizontal direction and 140 MPa compression in the vertical direction (see figure). Also, shear stresses of magnitude 60 MPa act in the directions shown.

Determine the stresses acting on an element oriented at a counterclockwise angle of 48° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.



Solution 7.2-4 **Plane stress (angle θ)**



$\sigma_x = 42$ MPa $\quad \sigma_y = -140$ MPa $\quad \tau_{xy} = -60$ MPa

$\theta = 48°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

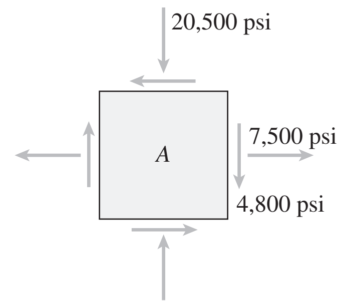
$$= -118.2 \text{ MPa} \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

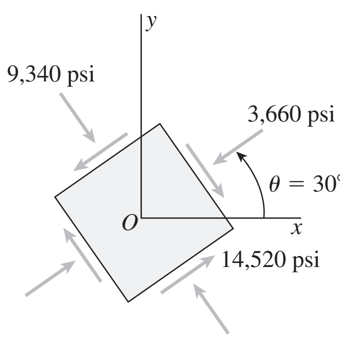
$$= -84.2 \text{ MPa} \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 20.2 \text{ MPa} \longleftarrow$$

Problem 7.2-5 Solve the preceding problem if the normal and shear stresses acting on element A are 7,500 psi, 20,500 psi, and 4,800 psi (in the directions shown in the figure) and the angle is 30° (counterclockwise).



Solution 7.2-5 **Plane stress (angle θ)**



$\sigma_x = 7{,}500$ psi $\quad \sigma_y = -20{,}500$ psi

$\tau_{xy} = -4{,}800$ psi

$\theta = 30°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -3{,}660 \text{ psi} \longleftarrow$$

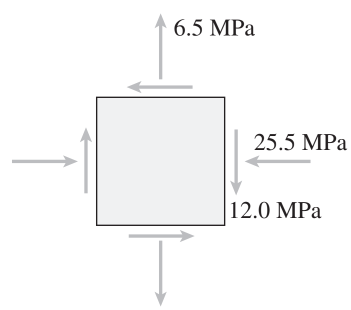
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -14,520 \text{ psi} \longleftarrow$$

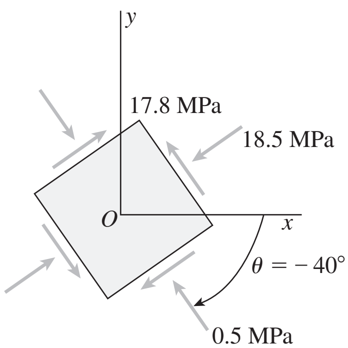
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -9{,}340 \text{ psi} \longleftarrow$$

Problem 7.2-6 An element in *plane stress* from the fuselage of an airplane is subjected to compressive stresses of magnitude 25.5 MPa in the horizontal direction and tensile stresses of magnitude 6.5 MPa in the vertical direction (see figure). Also, shear stresses of magnitude 12.0 MPa act in the directions shown.

Determine the stresses acting on an element oriented at a clockwise angle of 40° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.



Solution 7.2-6 **Plane stress (angle θ)**



$\sigma_x = -25.5$ MPa $\quad \sigma_y = 6.5$ MPa

$\tau_{xy} = -12.0$ MPa

$\theta = -40°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -0.5 \text{ MPa} \longleftarrow$$

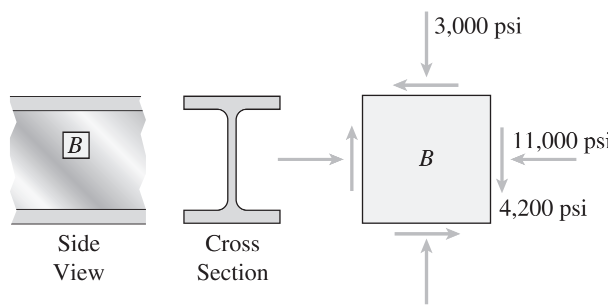
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -17.8 \text{ MPa} \longleftarrow$$

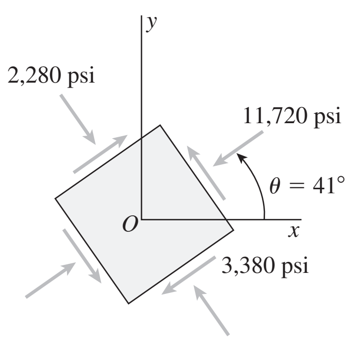
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -18.5 \text{ MPa} \longleftarrow$$

Problem 7.2-7 The stresses acting on element B in the web of a wide-flange beam are found to be 11,000 psi compression in the horizontal direction and 3,000 psi compression in the vertical direction (see figure). Also, shear stresses of magnitude 4,200 psi act in the directions shown.

Determine the stresses acting on an element oriented at a counterclockwise angle of 41° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.



Solution 7.2-7 **Plane stress (angle θ)**



$\sigma_x = -11{,}000$ psi $\quad \sigma_y = -3{,}000$ psi

$\tau_{xy} = -4{,}200$ psi

$\theta = 41°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

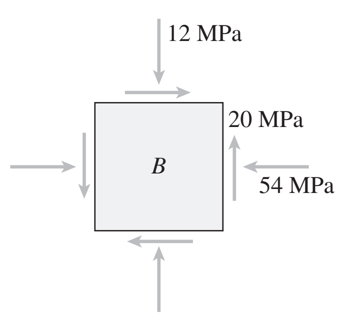
$$= -11{,}720 \text{ psi} \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

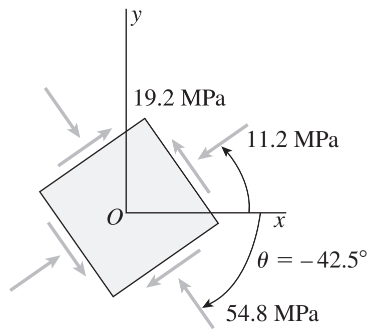
$$= 3{,}380 \text{ psi} \longleftarrow$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -2{,}280 \text{ psi} \longleftarrow$$

Problem 7.2-8 Solve the preceding problem if the normal and shear stresses acting on element B are 54 MPa, 12 MPa, and 20 MPa (in the directions shown in the figure) and the angle is 42.5° (clockwise).



Solution 7.2-8 **Plane stress (angle θ)**

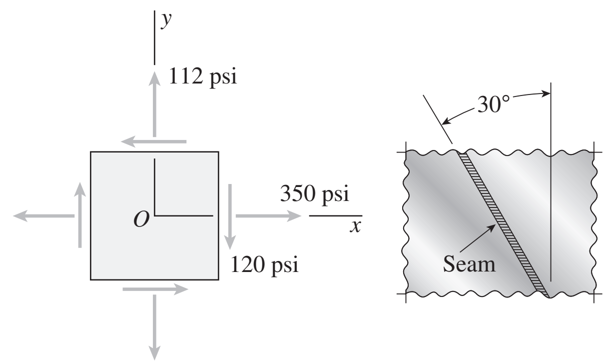


$\sigma_x = -54 \text{ MPa} \quad \sigma_y = -12 \text{ MPa} \quad \tau_{xy} = 20 \text{ MPa}$
$\theta = -42.5°$

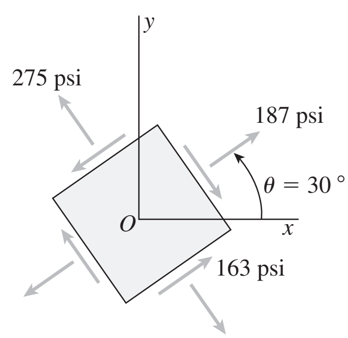
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= -54.8 \text{ MPa} \longleftarrow$$
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= -19.2 \text{ MPa} \longleftarrow$$
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -11.2 \text{ MPa} \longleftarrow$$

Problem 7.2-9 The polyethylene liner of a settling pond is subjected to stresses $\sigma_x = 350$ psi, $\sigma_y = 112$ psi, and $\tau_{xy} = -120$ psi, as shown by the plane-stress element in the first part of the figure.

Determine the normal and shear stresses acting on a seam oriented at an angle of 30° to the element, as shown in the second part of the figure. Show these stresses on a sketch of an element having its sides parallel and perpendicular to the seam.



Solution 7.2-9 **Plane stress (angle θ)**



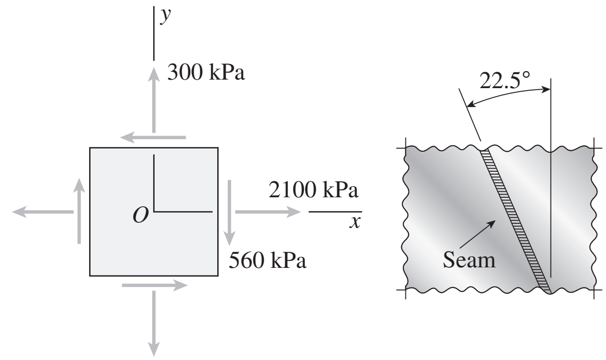
$\sigma_x = 350 \text{ psi} \quad \sigma_y = 112 \text{ psi} \quad \tau_{xy} = -120 \text{ psi}$
$\theta = 30°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= 187 \text{ psi} \longleftarrow$$
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= -163 \text{ psi} \longleftarrow$$
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 275 \text{ psi} \longleftarrow$$

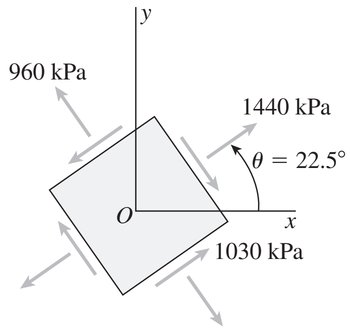
The normal stress on the seam equals 187 psi tension. $\longleftarrow$

The shear stress on the seam equals 163 psi, acting clockwise against the seam. $\longleftarrow$

Problem 7.2-10 Solve the preceding problem if the normal and shear stresses acting on the element are $\sigma_x = 2100$ kPa, $\sigma_y = 300$ kPa, and $\tau_{xy} = -560$ kPa, and the seam is oriented at an angle of 22.5° to the element (see figure).



Solution 7.2-10 **Plane stress (angle θ)**



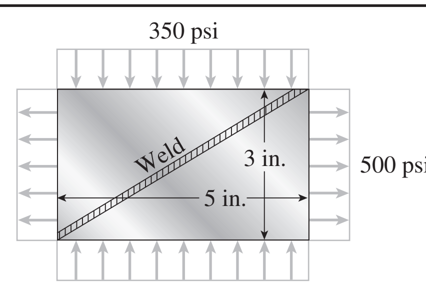
$\sigma_x = 2100$ kPa $\quad \sigma_y = 300$ kPa $\quad \tau_{xy} = -560$ kPa
$\theta = 22.5°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$
$$= 1440 \text{ kPa} \longleftarrow$$
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$
$$= -1030 \text{ kPa} \longleftarrow$$
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 960 \text{ kPa} \longleftarrow$$

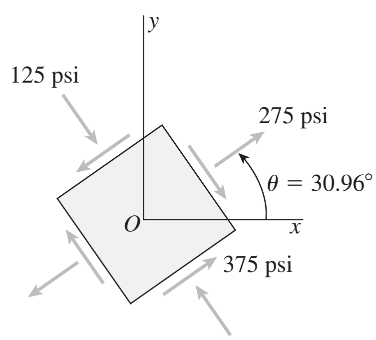
The normal stress on the seam equals 1440 kPa tension. ⟵
The shear stress on the seam equals 1030 kPa, acting clockwise against the seam. ⟵

Problem 7.2-11 A rectangular plate of dimensions 3.0 in. × 5.0 in. is formed by welding two triangular plates (see figure). The plate is subjected to a tensile stress of 500 psi in the long direction and a compressive stress of 350 psi in the short direction.

Determine the normal stress σ_w acting perpendicular to the line of the weld and the shear stress τ_w acting parallel to the weld. (Assume that the normal stress σ_w is positive when it acts in tension against the weld and the shear stress τ_w is positive when it acts counterclockwise against the weld.)



Solution 7.2-11 **Biaxial stress (welded joint)**

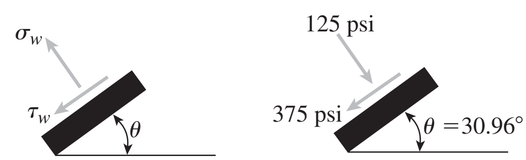


$\sigma_x = 500$ psi $\quad \sigma_y = -350$ psi $\quad \tau_{xy} = 0$

$$\theta = \arctan \frac{3 \text{ in.}}{5 \text{ in.}} = \arctan 0.6 = 30.96°$$

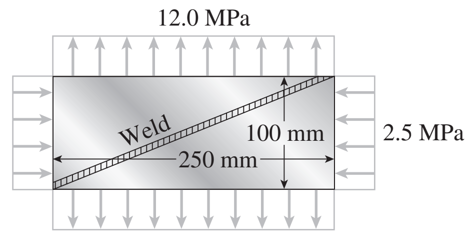
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$
$$= 275 \text{ psi}$$
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = -375 \text{ psi}$$
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -125 \text{ psi}$$

Stresses acting on the weld

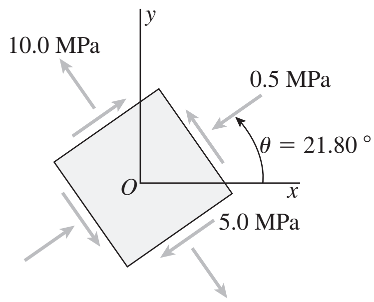


$$\sigma_w = -125 \text{ psi} \longleftarrow$$
$$\tau_w = 375 \text{ psi} \longleftarrow$$

Problem 7.2-12 Solve the preceding problem for a plate of dimensions 100 mm × 250 mm subjected to a compressive stress of 2.5 MPa in the long direction and a tensile stress of 12.0 MPa in the short direction (see figure).



Solution 7.2-12 **Biaxial stress (welded joint)**



$\sigma_x = -2.5$ MPa $\sigma_y = 12.0$ MPa $\tau_{xy} = 0$

$$\theta = \arctan \frac{100 \text{ mm}}{250 \text{ mm}} = \arctan 0.4 = 21.80°$$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -0.5 \text{ MPa}$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = 5.0 \text{ MPa}$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 10.0 \text{ MPa}$$

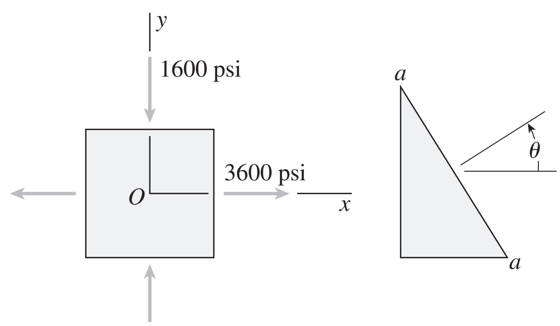
Stresses acting on the weld

$\sigma_w = 10.0$ MPa ⟵

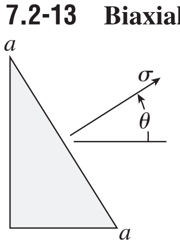
$\tau_w = -5.0$ MPa ⟵

Problem 7.2-13 At a point on the surface of a machine the material is in *biaxial stress* with $\sigma_x = 3600$ psi and $\sigma_y = -1600$ psi, as shown in the first part of the figure. The second part of the figure shows an inclined plane *aa* cut through the same point in the material but oriented at an angle θ.

Determine the value of the angle θ between zero and 90° such that no normal stress acts on plane *aa*. Sketch a stress element having plane *aa* as one of its sides and show all stresses acting on the element.



Solution 7.2-13 **Biaxial stress**



$\sigma_x = 3600$ psi
$\sigma_y = -1600$ psi
$\tau_{xy} = 0$

Find angle θ for $\sigma = 0$.
σ = normal stress on plane area

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 1000 + 2600\cos 2\theta \text{ (psi)}$$

For $\sigma_{x_1} = 0$, we obtain $\cos 2\theta = -\frac{1000}{2600}$

$\therefore \quad 2\theta = 112.62°$ and $\theta = 56.31°$ ⟵

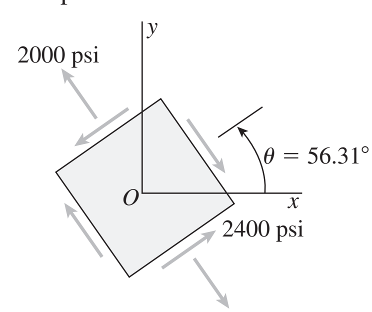
Stress element

$\sigma_{x_1} = 0 \quad \theta = 56.31°$

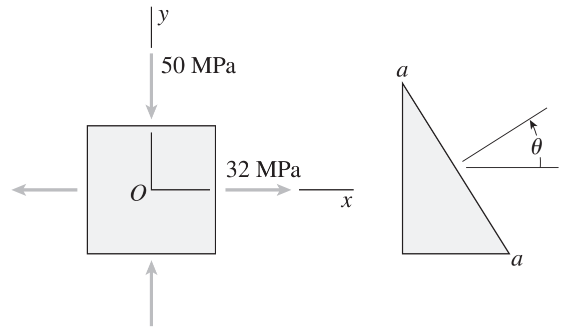
$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 2000$ psi ⟵

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

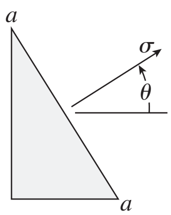
$= -2400$ psi ⟵



Problem 7.2-14 Solve the preceding problem for $\sigma_x = 32$ MPa and $\sigma_y = -50$ MPa (see figure).



Solution 7.2-14 **Biaxial stress**



$\sigma_x = 32$ MPa
$\sigma_y = -50$ MPa
$\tau_{xy} = 0$

Find angles θ for $\sigma = 0$.
σ = normal stress on plane area

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -9 + 41\cos 2\theta \text{ (MPa)}$$

For $\sigma_{x_1} = 0$, we obtain $\cos 2\theta = \frac{9}{41}$

$\therefore \quad 2\theta = 77.32°$ and $\theta = 38.66°$ ←

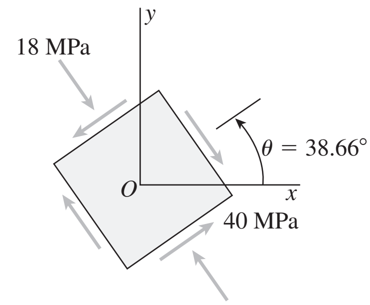
STRESS ELEMENT

$\sigma_{x_1} = 0 \quad \theta = 38.66°$

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -18$ MPa ←

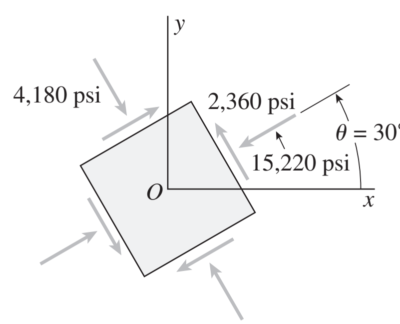
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -40 \text{ MPa}$$ ←



Problem 7.2-15 An element in *plane stress* from the frame of a racing car is oriented at a known angle θ (see figure). On this inclined element, the normal and shear stresses have the magnitudes and directions shown in the figure.

Determine the normal and shear stresses acting on an element whose sides are parallel to the xy axes; that is, determine σ_x, σ_y, and τ_{xy}. Show the results on a sketch of an element oriented at $\theta = 0°$.



Solution 7.2-15 **Plane stress**

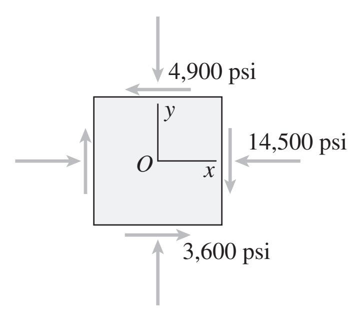
Transform from $\theta = 30°$ to $\theta = 0°$.
Let: $\sigma_x = -15{,}220$ psi, $\sigma_y = -4{,}180$ psi,
$\tau_{xy} = 2{,}860$ psi, and $\theta = -30°$.

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= -14{,}500 \text{ psi}$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = -3{,}600 \text{ psi}$$

$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -4{,}900 \text{ psi}$$



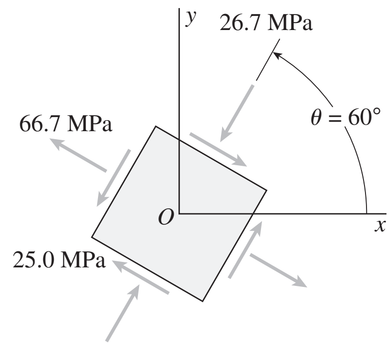
FOR $\theta = 0$:

$\sigma_x = \sigma_{x_1} = -14{,}500$ psi ←

$\sigma_y = \sigma_{y_1} = -4{,}900$ psi ←

$\tau_{xy} = \tau_{x_1y_1} = -3{,}600$ psi ←

Problem 7.2-16 Solve the preceding problem for the element shown in the figure.



Solution 7.2-16 Plane stress

Transform from $\theta = 60°$ to $\theta = 0°$.

Let: $\sigma_x = -26.7$ MPa, $\sigma_y = 66.7$ MPa, $\tau_{xy} = -25.0$ MPa, and $\theta = -60°$.

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 65 \text{ MPa}$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = -28 \text{ MPa}$$

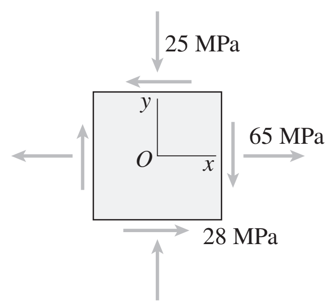
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = -25 \text{ MPa}$$

FOR $\theta = 0$:

$\sigma_x = \sigma_{x_1} = 65$ MPa ←

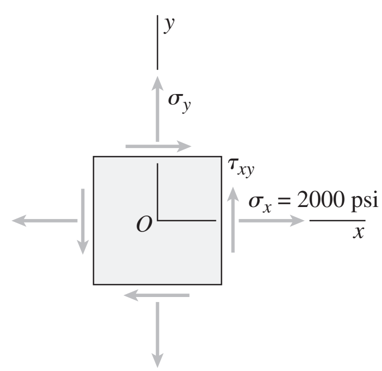
$\sigma_y = \sigma_{y_1} = -25$ MPa ←

$\tau_{xy} = \tau_{x_1y_1} = -28$ MPa ←



Problem 7.2-17 A plate in *plane stress* is subjected to normal stresses σ_x and σ_y and shear stress τ_{xy}, as shown in the figure. At counterclockwise angles $\theta = 40°$ and $\theta = 80°$ from the x axis the normal stress is 5000 psi tension.

If the stress σ_x equals 2000 psi tension, what are the stresses σ_y and τ_{xy}?



Solution 7.2-17 Plane stress

$\sigma_x = 2000$ psi $\sigma_y = ?$ $\tau_{xy} = ?$

At $\theta = 40°$ and $\theta = 80°$; $\sigma_{x_1} = 5000$ psi (tension)

Find σ_y and τ_{xy},

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

FOR $\theta = 40°$:

$\sigma_{x_1} = 5000$

$$= \frac{2000 + \sigma_y}{2} + \frac{2000 - \sigma_y}{2}\cos 80° + \tau_{xy}\sin 80°$$

or $\quad 0.41318\sigma_y + 0.98481\tau_{xy} = 3826.4 \text{ psi} \qquad (1)$

FOR $\theta = 80°$:

$\sigma_{x_1} = 5000$

$$= \frac{2000 + \sigma_y}{2} + \frac{2000 - \sigma_y}{2}\cos 160° + \tau_{xy}\sin 160°$$

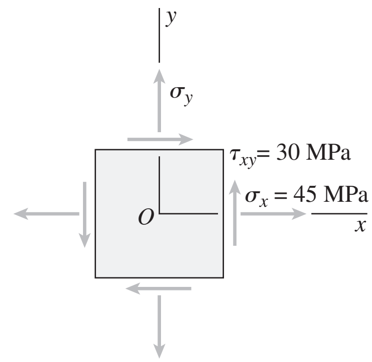
or $\quad 0.96985\sigma_y + 0.34202\tau_{xy} = 4939.7 \text{ psi} \qquad (2)$

SOLVE EQS. (1) AND (2):

$\sigma_y = 4370$ psi $\tau_{xy} = 2050$ psi ←

Problem 7.2-18 The surface of an airplane wing is subjected to *plane stress* with normal stresses σ_x and σ_y and shear stress τ_{xy}, as shown in the figure. At a counterclockwise angle $\theta = 30°$ from the x axis the normal stress is 35 MPa tension, and at an angle $\theta = 50°$ it is 10 MPa compression.

If the stress σ_x equals 100 MPa tension, what are the stresses σ_y and τ_{xy}?



Solution 7.2-18 Plane stress

$\sigma_x = 100$ MPa $\quad \sigma_y = ?\quad \tau_{xy} = ?$

At $\theta = 30°$, $\sigma_{x_1} = 35$ MPa (tension)

At $\theta = 50°$, $\sigma_{x_1} = -10$ MPa (compression)

Find σ_y and τ_{xy}

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

FOR $\theta = 30°$:

$$\sigma_{x_1} = 35 = \frac{100 + \sigma_y}{2} + \frac{100 - \sigma_y}{2}\cos 60° + \tau_{xy}\sin 60°$$

or $\quad 0.25\sigma_y + 0.86603\tau_{xy} = -40$ MPa $\qquad$ (1)

FOR $\theta = 50°$:

$$\sigma_{x_1} = -10 = \frac{100 + \sigma_y}{2} + \frac{100 - \sigma_y}{2}\cos 100° + \tau_{xy}\sin 100°$$

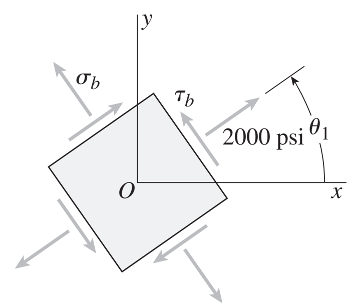
or $\quad 0.58682\sigma_y + 0.98481\tau_{xy} = -51.318$ MPa $\qquad$ (2)

SOLVE EQS. (1) AND (2):

$\sigma_y = -19.3$ MPa $\qquad \tau_{xy} = -40.6$ MPa ⟵

Problem 7.2-19 At a point in a structure subjected to *plane stress*, the stresses are $\sigma_x = -4000$ psi, $\sigma_y = 2500$ psi, and $\tau_{xy} = 2800$ psi (the sign convention for these stresses is shown in Fig. 7-1). A stress element located at the same point in the structure, but oriented at a counterclockwise angle θ_1 with respect to the x axis, is subjected to the stresses shown in the figure (σ_b, τ_b, and 2000 psi).

Assuming that the angle θ_1 is between zero and 90°, calculate the normal stress σ_b, the shear stress τ_b, and the angle θ_1.



Solution 7.2-19 Plane stress

$\sigma_x = -4000$ psi $\quad \sigma_y = 2500$ psi $\quad \tau_{xy} = 2800$ psi

FOR $\theta = \theta_1$:

$\sigma_{x_1} = 2000$ psi $\quad \sigma_{y_1} = \sigma_b \quad \tau_{xy} = \tau_b$

Find σ_b, τ_b, and θ_1

STRESS σ_b

$\sigma_b = \sigma_x + \sigma_y - 2000$ psi $= -3500$ psi ⟵

ANGLE θ_1

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

2000 psi $= -750 - 3250\cos 2\theta_1 + 2800\sin 2\theta_1$

or $-65\cos 2\theta_1 + 56\sin 2\theta_1 - 65 = 0$

Solve numerically:

$2\theta_1 = 89.12°$ $\quad \theta_1 = 44.56°$ ⟵

SHEAR STRESS τ_b

$$\tau_b = \tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta_1 + \tau_{xy}\cos 2\theta_1$$

$= 3290$ psi ⟵

Principal Stresses and Maximum Shear Stresses

When solving the problems for Section 7.3, consider only the in-plane stresses (the stresses in the xy plane).

Problem 7.3-1 An element in plane stress is subjected to stresses $\sigma_x = 6500$ psi, $\sigma_y = 1700$ psi, and $\tau_{xy} = 2750$ psi (see the figure for Problem 7.2-1).

Determine the principal stresses and show them on a sketch of a properly oriented element.

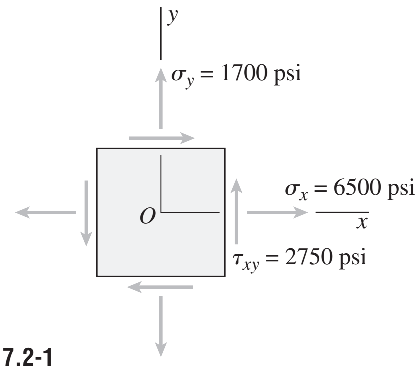


Figure 7.2-1

Solution 7.3-1 Principal stresses

$\sigma_x = 6500$ psi $\quad \sigma_y = 1700$ psi $\quad \tau_{xy} = 2750$ psi

PRINCIPAL STRESSES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.1458$$

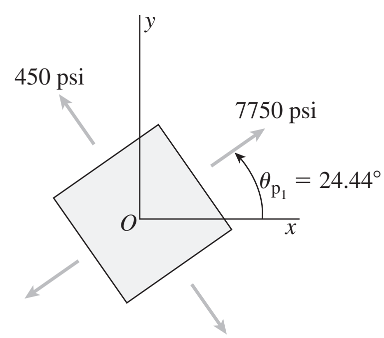
$2\theta_p = 48.89°$ and $\theta_p = 24.44°$
$2\theta_p = 228.89°$ and $\theta_p = 114.44°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 48.89°$: $\sigma_{x_1} = 7750$ psi

For $2\theta_p = 228.89°$: $\sigma_{x_1} = 450$ psi

Therefore, $\sigma_1 = 7750$ psi and $\theta_{p_1} = 24.44°$
$\sigma_2 = 450$ psi and $\theta_{p_2} = 114.44°$ ←



Problem 7.3-2 An element in plane stress is subjected to stresses $\sigma_x = 80$ MPa, $\sigma_y = 52$ MPa, and $\tau_{xy} = 48$ MPa (see the figure for Problem 7.2-2).

Determine the principal stresses and show them on a sketch of a properly oriented element.

Solution 7.3-2 Principal stresses

$\sigma_x = 80$ MPa $\quad \sigma_y = 52$ MPa $\quad \tau_{xy} = 48$ MPa

PRINCIPAL STRESSES

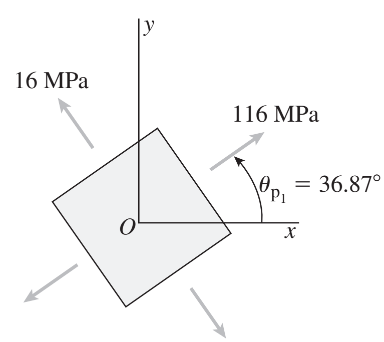
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 3.429$$

$2\theta_p = 73.74°$ and $\theta_p = 36.87°$
$2\theta_p = 253.74°$ and $\theta_p = 126.87°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 73.74°$: $\sigma_{x_1} = 116$ MPa
For $2\theta_p = 253.74°$: $\sigma_{x_1} = 16$ MPa

Therefore, $\sigma_1 = 116$ MPa and $\theta_{p_1} = 36.87°$
$\sigma_2 = 16$ MPa and $\theta_{p_2} = 126.87°$ ←



Problem 7.3-3 An element in plane stress is subjected to stresses $\sigma_x = -9{,}900$ psi, $\sigma_y = -3{,}400$ psi, and $\tau_{xy} = 3{,}600$ psi (see the figure for Problem 7.2-3).

Determine the principal stresses and show them on a sketch of a properly oriented element.

Solution 7.3-3 Principal stresses

$\sigma_x = -9900$ psi $\quad \sigma_y = -3400$ psi $\quad \tau_{xy} = 3600$ psi

PRINCIPAL STRESSES

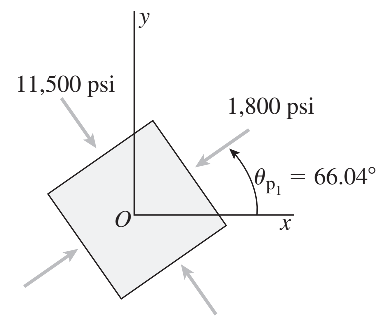
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.1077$$

$2\theta_p = -47.92°$ and $\theta_p = -23.96°$
$2\theta_p = 132.08°$ and $\theta_p = 66.04°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -47.92°$: $\sigma_{x_1} = -11{,}500$ psi
For $2\theta_p = 132.08°$: $\sigma_{x_1} = -1{,}800$ psi

Therefore, $\sigma_1 = -1{,}800$ psi and $\theta_{p_1} = 66.04°$
$\sigma_2 = -11{,}500$ psi and $\theta_{p_2} = -23.96°$ ←



Problem 7.3-4 An element in plane stress is subjected to stresses $\sigma_x = 42$ MPa, $\sigma_y = -140$ MPa, and $\tau_{xy} = -60$ MPa (see the figure for Problem 7.2-4).

Determine the principal stresses and show them on a sketch of a properly oriented element.

Solution 7.3-4 Principal stresses

$\sigma_x = 42$ MPa $\quad \sigma_y = -140$ MPa
$\tau_{xy} = -60$ MPa

PRINCIPAL STRESSES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.6593$$

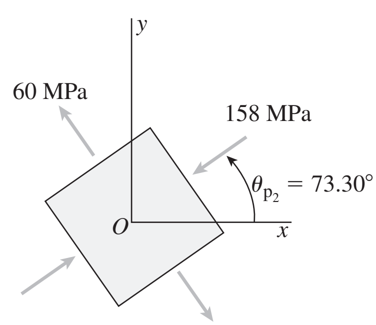
$2\theta_p = -33.40°$ and $\theta_p = -16.70°$
$2\theta_p = 146.60°$ and $\theta_p = 73.80°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -33.40°$: $\sigma_{x_1} = 60$ MPa

For $2\theta_p = 146.60°$: $\sigma_{x_1} = -158$ MPa

Therefore, $\sigma_1 = 60$ MPa and $\theta_{p_1} = -16.70°$
$\sigma_2 = -158$ MPa and $\theta_{p_2} = 73.30°$ ←



Problem 7.3-5 An element in plane stress is subjected to stresses $\sigma_x = 7{,}500$ psi, $\sigma_y = -20{,}500$ psi, and $\tau_{xy} = -4{,}800$ psi (see the figure for Problem 7.2-5).

Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

Solution 7.3-5 Maximum shear stresses

$\sigma_x = 7{,}500$ psi $\quad \sigma_y = -20{,}500$ psi
$\tau_{xy} = -4{,}800$ psi

PRINCIPAL ANGLES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.3429$$

$2\theta_p = -18.92°$ and $\theta_p = -9.46°$
$2\theta_p = 161.08°$ and $\theta_p = 80.54°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

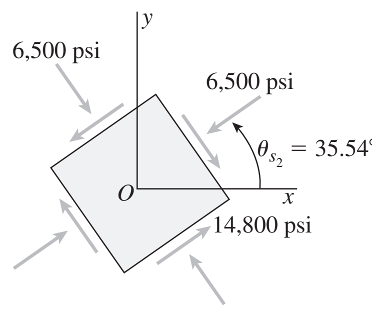
For $2\theta_p = -18.92°$: $\quad \sigma_{x_1} = 8{,}300$ psi
For $2\theta_p = 161.08°$: $\quad \sigma_{x_1} = -21{,}300$ psi
Therefore, $\theta_{p_1} = -9.46°$

MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 14{,}800 \text{ psi}$$

$$\left.\begin{aligned}\theta_{s_1} &= \theta_{p_1} - 45° = -54.46° \text{ and } \tau = 14{,}800 \text{ psi}\\ \theta_{s_2} &= \theta_{p_1} + 45° = 35.54° \text{ and } \tau = -14{,}800 \text{ psi}\end{aligned}\right\} \longleftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -6{,}500 \text{ psi} \quad \longleftarrow$$



Problem 7.3-6 An element in plane stress is subjected to stresses $\sigma_x = -25.5$ MPa, $\sigma_y = 6.5$ MPa, and $\tau_{xy} = -12.0$ MPa (see the figure for Problem 7.2-6).

Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

Solution 7.3-6 Maximum shear stresses

$\sigma_x = -25.5$ MPa $\quad \sigma_y = 6.5$ MPa
$\tau_{xy} = -12.0$ MPa

PRINCIPAL ANGLES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.7500$$

$2\theta_p = 36.87°$ and $\theta_p = 18.43°$
$2\theta_p = 216.87°$ and $\theta_p = 108.43°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

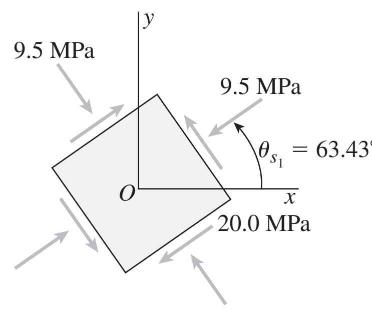
For $2\theta_p = 36.87°$: $\quad \sigma_{x_1} = -29.5$ MPa
For $2\theta_p = 216.87°$: $\quad \sigma_{x_1} = 10.5$ MPa
Therefore, $\theta_{p_1} = 108.4°$

MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 20.0 \text{ MPa}$$

$$\left.\begin{aligned}\theta_{s_1} &= \theta_{p_1} - 45° = 63.48° \text{ and } \tau = 20.0 \text{ MPa}\\ \theta_{s_2} &= \theta_{p_1} + 45° = 153.43° \text{ and } \tau = -20.0 \text{ MPa}\end{aligned}\right\} \longleftarrow$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -9.5 \text{ MPa} \quad \longleftarrow$$



Problem 7.3-7 An element in plane stress is subjected to stresses $\sigma_x = -11{,}000$ psi, $\sigma_y = -3{,}000$ psi, and $\tau_{xy} = -4200$ psi (see the figure for Problem 7.2-7).

Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

Solution 7.3-7 Maximum shear stresses

$\sigma_x = -11{,}000$ psi $\quad \sigma_y = -3{,}000$ psi

$\tau_{xy} = -4{,}200$ psi

PRINCIPAL ANGLES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.0500$$

$2\theta_p = 46.40°$ and $\theta_p = 23.20°$

$2\theta_p = 226.40°$ and $\theta_p = 113.20°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 46.40°$: $\quad \sigma_{x_1} = -12{,}800$ psi

For $2\theta_p = 226.40°$: $\quad \sigma_{x_1} = -1{,}200$ psi

Therefore, $\theta_{p_1} = 113.20°$

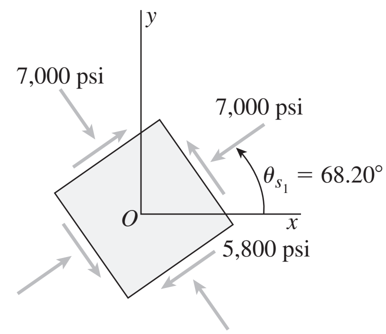
MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5{,}800 \text{ psi}$$

$\theta_{s_1} = \theta_{p_1} - 45° = 68.20°$ and $\tau = 5{,}800$ psi

$\theta_{s_2} = \theta_{p_1} + 45° = 158.20°$ and $\tau = -5{,}800$ psi ←

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -7{,}000 \text{ psi} \quad \leftarrow$$



Problem 7.3-8 An element in plane stress is subjected to stresses $\sigma_x = -54$ MPa, $\sigma_y = -12$ MPa, and $\tau_{xy} = 20$ MPa (see the figure for Problem 7.2-8).

Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

Solution 7.3-8 Maximum shear stresses

$\sigma_x = -54$ MPa $\quad \sigma_y = -12$ MPa

$\tau_{xy} = 20$ MPa

PRINCIPAL ANGLES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.9524$$

$2\theta_p = -43.60°$ and $\theta_p = -21.80°$

$2\theta_p = 136.40°$ and $\theta_p = 68.20°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = -43.60°$: $\quad \sigma_{x_1} = -162$ MPa

For $2\theta_p = 136.40°$: $\quad \sigma_{x_1} = -4.0$ MPa

Therefore, $\theta_{p_1} = 68.20°$

MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 29.0 \text{ MPa}$$

$\theta_{s_1} = \theta_{p_1} - 45° = 23.20°$ and $\tau = 29.0$ MPa

$\theta_{s_2} = \theta_{p_1} + 45° = 113.20°$ and $\tau = -29.0$ MPa ←

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -33.0 \text{ MPa} \quad \leftarrow$$